Accordingly, $N=mg(p-\frac{v_0^2}{g})=\frac{m}{p}(pg-v^2)$.

If $v_0^2=pg$ the normal reaction is always zero.

If $v_0^2>pg$ the particle leaves the curve at once.

If $v_0^2<pg$ the particle is constrained to move on the curve.

Then the particle either—

1°. Describes the curve freely without leaving it ; or,

2°. Leaves the curve at the beginning of the motion ; or,

3°. Describes the curve under constraint without leaving it.

Also solved by *G. B. M. ZERR*, and the *PROPOSER*.

DIOPHANTINE ANALYSIS.

75. Proposed by CHARLES CARROLL CROSS. Whaleyville, Va.

Arrange the consecutive integers 1 to n^2 as a magic square, where n is odd. Apply when $n=9$.

Solution by M. A. GRUBER, A. M., War Department, Washington, D. C.

This solution for odd magic squares is adapted from a similar solution found in an article on "Evening Entertainments."

We divide a large square into as many little spaces as there are numbers in the magic square.

We conceive a similar large square, similarly divided, bordering on the right, another on the bottom, another on the left, and a fourth meeting the right bottom corner of the given square.

We fill in the numbers consecutively, commencing with unity, *from left diagonally down to right, beginning with the space immediately below the center space.*

When reaching the limit of the given square, we continue the next number into the "bordering square." This number will then be placed in the *corresponding space of the given square.*

When, in filling in the numbers, we meet a space that is already occupied, we take the space *next diagonally down left from this occupied space*, and continue as before.

Take a magic square having 9 numbers on a side. There will then be 9^2, or 81 spaces in the square, to be filled with the numbers 1 to 81 inclusive.

Beginning with the space next below the center space, and filling in diagonally down to right, we find 4 reaches the limit of the square. Whence 5 occupies, in the "bordering square," the right upper corner. We now put 5 in the right upper corner of the given square, and find it at the limit of the square. Whence 6 occupies, in the "bordering square," the space next below the left upper corner. We now place 6 in the corresponding space of the given square, and proceed 6, 7, 8, 9, when we meet the space occupied by 1. We then put 10 in the space next diagonally down left from 1, and proceed 10, 11, 12, etc., until all the spaces are filled.

		37	78	29	70	21	62	13	54	5		
	46	6	38	79	30	71	22	63	14	46	6	
		47	7	39	80	31	72	23	55	15	47	
		16	48	8	40	81	32	64	24	56	16	
		57	17	49	9	41	73	33	65	25	57	
		26	58	18	50	1	42	74	34	66	26	
		67	27	59	10	51	2	43	75	35	67	
		36	68	19	60	11	52	3	44	76	36	
		77	28	69	20	61	12	53	4	45	77	
		37	78	29	70	21	62	13	54	5	46	

AVERAGE AND PROBABILITY.

79. Proposed by the late ENOCH BEERY SEITZ.

Two equal spheres touch each other externally. If a point be taken at random within each sphere, show that (1) the chance that the distance between the points is less than the diameter of either sphere is 13/35, and (2) the average distance between them is $11/5r$. [This is problem 5835, *Educational Times*, of London.]

Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Mathematics and Science, Chester High School, Chester, Pa.

(1) Let A, B be the centers and C the point of contact of the two spheres, each radius r.

From any point P in DC with a radius$=2r$ describe a sphere cutting B in Q, R. From B as a center with a radius BP describe a sphere cutting A in K, M. If P is the first point, the second point must fall within the double-convex lens $CQRC$. P may fall anywhere on the zone KPM and the second point must fall in a section of B equal to the double-convex lens $CQRC$.

From P as center with a radius $PS<2r$ but$>PC$, draw the zone SLT. Let $DP=x$, $PS=y$, area of zone $KPM=2\pi.BP.PG$, area of zone $SLT=2\pi.PS.HL$.

$BP=3r-x$, $AG=r-x-PG$, $BG=3r-x-PG$, $PS=y$, $BH=3r-x-y+HL$, $PH=y-HL$.

$KG^2=r^2-(r-x-PG)^2=(3r-x)^2-(3r-x-PG)^2$.

$\therefore PG=x(2r-x)/4r$.

$SH^2=r^2-(3r-x-y+HL)^2=y^2-(y-HL)^2$.

$\therefore HL=[r^2-(3r-x-y)^2]/2(3r-x)$.

$\therefore$ Area of zone $KPM=(\pi x/2r)(3r-x)(2r-x)$.

Area of zone $SLT=[\pi y/(3r-x)][r^2-(3r-x-y)^2]$.

Let $p=$chance, $\triangle=$average distance.